PART II: WORKOUT PROBLEMS. Show all the necessary steps clearly and neatly on the space provided. (5 points each)

1. Use Laplace transformation method to solve the IVP:

$$y' + \int_0^t y(\tau)\,d\tau = f(t);\, y(0) = 0, where\ f(t) = \begin{cases} 0, 0 \le t \le 2 \\ t,\ \ t \ge 2 \end{cases}$$

The function f interms of unit step function u:

$f(t) = t\,u(t-2)$ 0.5

The given DE is:

$y' + \int_0^t y(\tau)\,d\tau = t\,u(t-2)$ 0.5

$\Rightarrow \mathcal{L}(y') + \mathcal{L}(y * 1) = \mathcal{L}(t\,u(t-2))$ 1

$\Rightarrow s\mathcal{L}(y) - y(0) + \mathcal{L}(y)\mathcal{L}(1) = \mathcal{L}[(t-2)u(t-2)] + 2\mathcal{L}(u(t-2))$

$\Rightarrow s\mathcal{L}(y) + \mathcal{L}(y)\frac{1}{s} = \frac{e^{-2s}}{s^2} + \frac{2e^{-2s}}{s}$

$\Rightarrow (s + \frac{1}{s})\mathcal{L}(y) = \frac{e^{-2s}}{s^2} + \frac{2e^{-2s}}{s}$ 1

$\Rightarrow \frac{(s^2+1)}{s}\mathcal{L}(y) = \frac{e^{-2s}}{s^2} + \frac{2e^{-2s}}{s}$

$\Rightarrow \mathcal{L}(y) = \frac{1}{s(s^2+1)}e^{-2s} + \frac{2}{s^2+1}e^{-2s} = \frac{e^{-2s}}{s} - \frac{s}{s^2+1}e^{-2s} + \frac{2e^{-2s}}{s^2+1}$ 1

$\Rightarrow y = \mathcal{L}^{-1}\left(\frac{e^{-2s}}{s}\right) - \mathcal{L}^{-1}\left(\frac{s}{s^2+1}e^{-2s}\right) + \mathcal{L}^{-1}\left(\frac{2e^{-2s}}{s^2+1}\right)$

$\Rightarrow y(t) = u(t-2) - u(t-2)\cos(t-2) + 2u(t-2)\sin(t-2)$ 1